MATH 161 HOMEWORK 1

1. Suppose A is a set. Let $B = \{x \in A : x \notin x\}$. [Of course B exists and is unique by the axioms of selection (i.e., comprehension) and extension.] Prove that B is NOT an element of A.

2. Use (1) to prove that there is no universal set. (There is no set U such that $x \in U$ for every x.)

3. Let A be a set. Prove that $\{x : x \notin A\}$ cannot be a set.

4. A set S is called a **singleton** if and only if $S = \{a\}$ for some a. Prove that there does not exist a set that contains every singleton.

5. Let $S(x)$ be a sentence. Suppose there are some x's for which $S(x)$ is true. Prove that

$$\{y : y \in x \text{ for every } x \text{ such that } S(x) \text{ is true}\}$$

is a set.

6. Let $[a, b]$ be the set $\{a, \{b\}\}$. Explain why this would not be a good definition of ordered pair.

7. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ for all sets A, B, and C. [Here $X \times Y$ is the "Cartesian product" of X and Y: it is the set of all ordered pairs (x, y) such that $x \in X$ and $y \in Y$. We will prove that $X \times Y$ exists in class on Tuesday.]

8. Some set theory books use weaker forms of our axioms, namely:

(i) Weak existence: There exists a set.
(ii) Extension (same as in our text.)
(iii) Comprehension (same is in our text).
(iv) Weak pairs: For every A and B, there exists a set C such that $A \in C$ and $B \in C$.
(v) Weak unions: for every set S, there exists a set T with the following property: for every x, if $x \in A$ for some $A \in S$, then $s \in T$.
(vi) Weak power set: for every set S, there is a set P with the following property: every subset of S is an element of P.

Explain why these axioms imply the "strong" versions given in the Hrbacek-Jech book (i.e., the versions given in class.)

NOTE: it is not true that each "weak" axiom implies the corresponding "strong" axiom. (It does not.) The claim is that the axioms (i)-(vi), taken together, imply the strong versions.