MATH 161 HOMEWORK 6 (DUE THUR, FEB 24 IN CLASS)

1. Let α be an ordinal. Show that α is a successor ordinal if and only if α has a greatest element.

2. Let α be an ordinal. Show that α is a limit ordinal if and only if $\cup\alpha = \alpha$.

3. Let $(I, <_I)$ be a well-ordered set. Suppose also that for each $i \in I$, there is a well-ordered set A_i and a well-ordering $<_i$ of A_i. Let $B = \cup_{i \in I}(\{i\} \times A_i)$, and define a linear order $<$ on B as follows:

$$(i, x) < (j, y)$$

if and only if:

$$i <_I j, \text{ or}$$
$$i = j \text{ and } x <_i y.$$

Prove that $<$ is a well-ordering of B.

[You do not need to include a proof that $<$ is indeed a linear order on B, but be sure you understand why it is one. Just prove that every nonempty subset of B has a least element.]

4. Let $(W, <)$ be a well-ordered set. Show that there is no function $f : \mathbf{N} \to W$ such that $f(x+1) < f(x)$ for every x. (Or, equivalently, show that if $f : \mathbf{N} \to W$, then $f(x+1) \geq f(x)$ for some x.)

5. You play a game of solitaire as follows. You begin with any number of quarters, dimes, nickels, and pennies. Each day, you are required to exchange one of your coins for any number of coins of smaller denominations. (For example, you could trade one dime for one billion nickes and one trillion pennies.) Prove that you must run out of coins in a finite number of days.

6. Let X be any set. Prove that there is a transitive set Y such that $X \subset Y$. Note: this problem would have been impossible two weeks ago.

7. Let V be the set of real numbers of the form $n - \frac{1}{m}$ where $n \in \mathbf{N}$ and $m \in \mathbf{N}$ and $m > 0$. Find an isomorphism from $(V, <)$ to an ordinal number.