MATH 161 SAMPLE MIDTERM QUESTIONS

1. Complete the following definitions.
(a). An **inductive set** is a set X such that
(b). The ordered pair $(x, y) =$
(c). A **relation** from A to B is
(d). A **natural number** is a set x such that
(e.) Let x be any set. The **successor** of x is
(f.) A **transitive set** is a set S such that

2. Prove carefully that there is no set S such that $x \in S$ for every x. Be sure you mention each axiom that you use.

3. Recall that addition of natural numbers is defined so that $n + 0 = n$ and $n + S(m) = S(n + m)$. Prove from the definition that $0 + n = n$ for every natural number n. [You should prove this "from scratch", i.e., without using facts about addition, subtraction, etc.]

4. Show that $\mathbf{N} \times \mathbf{N}$ is $\leq$ to $\mathbf{N}$ in cardinality by giving an example of a function $f : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ that is one-to-one.

5. Prove that $2^{\mathbf{N}}$ and $\mathcal{P}(\mathbf{N})$ have the same cardinality by giving a bijection from one to the other.

6. Prove that $\mathcal{P}(\mathbf{N})$ and $\mathcal{P}(\mathbf{N})^{\mathbf{N}}$ have the same cardinality. [Don't just say "because $\mathbf{R}$ and $\mathbf{R}^{\mathbf{N}}$ have the same cardinality".]

7. Let A be a set. Prove that there is a set B such that

$$x \in B \text{ if and only if } x = \mathcal{P}(y) \text{ for some } y \in A.$$

8. Consider two objects O and I (with $O \neq I$), which we will call "pseudosets". If x and y are pseudosets, say that x is a pseudoelement of y (and write $x \boxed{\in} y$) if and only if $y = I$.

Which of the axioms of set theory are true if we interpret "set" to mean "pseudoset" and "element" to mean "pseudoelement"? You may skip the axiom of infinity and the axiom of choice.

9. Prove that if S is a transitive set, then the power set of S is also a transitive set.

10. State the Cantor-Bernstein Theorem.